Programming with dependent types

Seminar

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Contents

A dependent type is a type
that depends on a value.

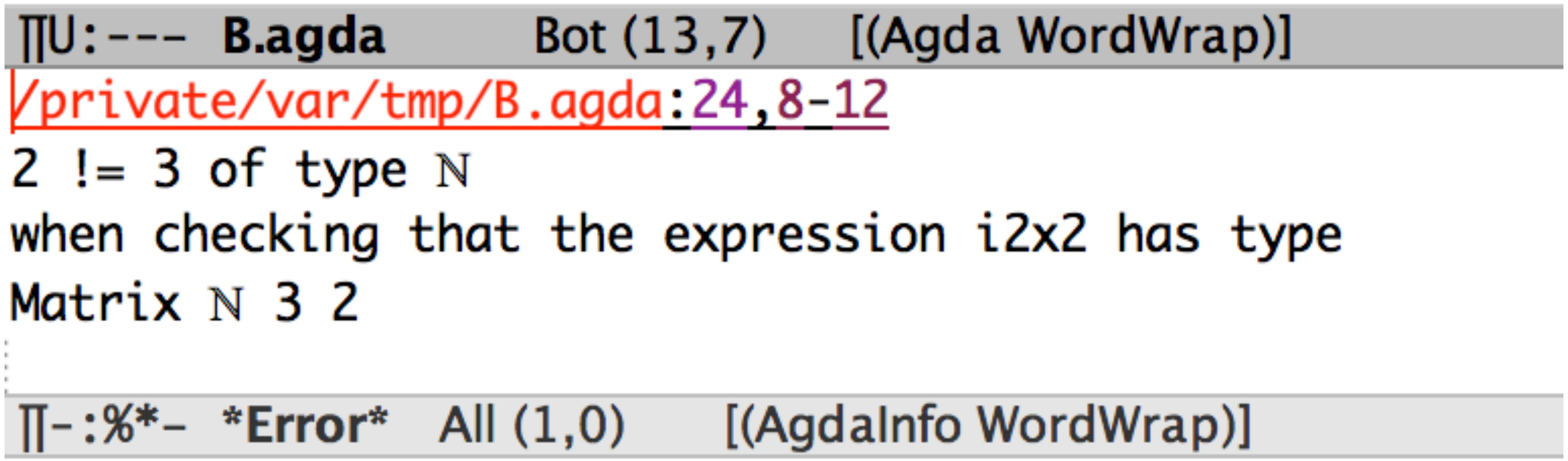
–Wikipedia

```
Matrix : Set → ℕ → ℕ → Set
Matrix A m n = Vec (Vec A n) m

i2x2 : Matrix ℕ 2 2
i2x2 = (1 ∷ 0 ∷ []) ∷
       (0 ∷ 1 ∷ []) ∷ []
```

Types that depends on values

```
m3x2 : Matrix ℕ 3 2
m3x2 = i2x2
```



```
∏U:---  B.agda        Bot (13,7)     [(Agda WordWrap)]
/private/var/tmp/B.agda:24,8-12
2 != 3 of type ℕ
when checking that the expression i2x2 has type
Matrix ℕ 3 2

∏-:%*-  *Error*   All (1,0)      [(AgdaInfo WordWrap)]
```

Types that depends on values

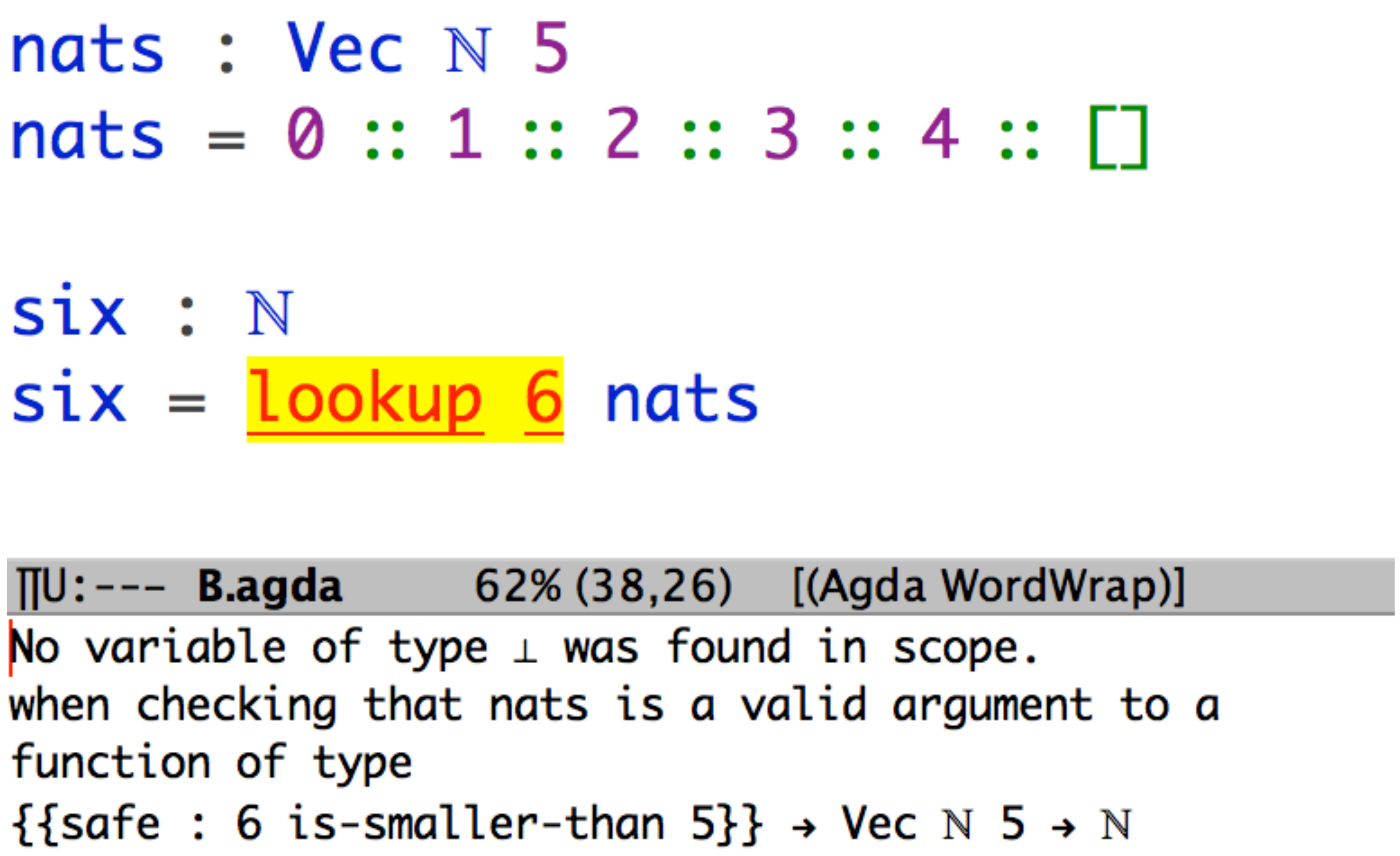
```
lookup : ∀ {A : Set} {n : ℕ} →
  (i : ℕ) →
  {{safe : i is-smaller-than n}} →
  Vec A n → A
```

Types that depends on values

```
nats : Vec ℕ 5
nats = 0 :: 1 :: 2 :: 3 :: 4 :: []

two : ℕ
two = lookup 2 nats
```

Types that depends on values


```
nats : Vec ℕ 5
nats = 0 ∷ 1 ∷ 2 ∷ 3 ∷ 4 ∷ []

six : ℕ
six = lookup 6 nats
```

```
-UU:---  B.agda       62% (38,26)    [(Agda WordWrap)]
No variable of type ⊥ was found in scope.
when checking that nats is a valid argument to a
function of type
{{safe : 6 is-smaller-than 5}} → Vec ℕ 5 → ℕ
```

Types that depends on values

Types *are* values

```
Matrix : Set → ℕ → ℕ → Set
Matrix A m n = Vec (Vec A n) m
```

Types *are* values

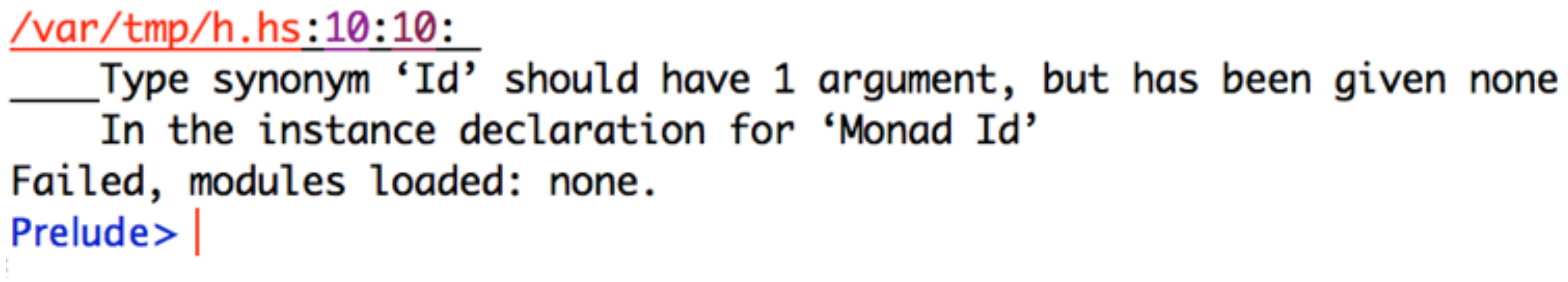
```
Id : Set → Set
Id A = A
```

Types *are* values

```haskell
-- Haskell
type Id a = a

instance Monad Id where
  return = \ x -> x
  (>>=)  = \ x f -> f x
```

Types *are* values



```
/var/tmp/h.hs:10:10:
    Type synonym ‘Id’ should have 1 argument, but has been given none
    In the instance declaration for ‘Monad Id’
Failed, modules loaded: none.
Prelude>
```

Types *are* values

```haskell
-- Haskell
newtype Id a = Id { runId :: a }

instance Monad Id where
  return = \ x -> Id x
  (>>=)  = \ x f -> f (runId x)
```

Types *are* values

I don't think we really expected newtypes to be quite so ubiquitous in this kind of way. The introduction of newtypes in typeclasses was much more fruitful than I'd at all expected when I designed the language.

–Simon Peyton Jones

```
-- Agda
Id : Set → Set
Id A = A

IdMonad : RawMonad Id
IdMonad = record
  { return = λ x → x
  ; _>>=_  = λ x f → f x
  }
```

Types *are* values

The Damas-Milner approach to type inference is alive and well and working harder than ever, even though we have dispensed with the shackles on programming which allow it to be complete.

–Altenkirch et al.

Organization

- Seminar website: ps.informatik.uni-tuebingen.de/teaching/ws15/pdt/
- Every week, somebody teaches everybody else something and leaves some homework exercises.
- First 4 topics are fixed, the rest are up to the presenter.
- There is a small individual programming project at the end.

Agda and Idris

- Very similar languages
- Choose one, both, or neither

Agda and Idris

- Agda 2.4.2.3 has a robust front-end and a questionable back-end.
- Idris 0.9.18.1 has a buggy front-end and a reasonable back-end.

Agda

- Fun interactive development environment (emacs)
- Reliable type inference and pattern matching
- Faster than computing on paper (usually)
- Little to no support for impure effects like IO or random number generation

Idris

- Shouldn’t be slower than conventional languages by much more than a constant factor
- Designed with systems programming in mind, comes with an effect system
- Gets confused by complicated pattern-matching
- Type inference is shaky, don’t rely on it

If we wanted, we could shut these machines down.

–Neo

```
-- Agda
{-# NON_TERMINATING #-}
loop : ⊥
loop = loop


-- Idris
partial
loop : Void
loop = loop
```

Dirty tricks

```
-- Agda
open import
  Relation.Binary.PropositionalEquality
open import
  Relation.Binary.PropositionalEquality.TrustMe
badcast : Bool → String
badcast x = subst (λ A → A) trustMe x


-- Idris
badcast : Bool -> String
badcast x = believe_me x
```

Dirty tricks

Homework

1. Install Idris
2. Install Agda
3. Install Agda standard library
4. Write hello-world programs
5. Email me:
    - Your experience with dependent types
    - The topics you are most interested in
    - Your hello-world program code